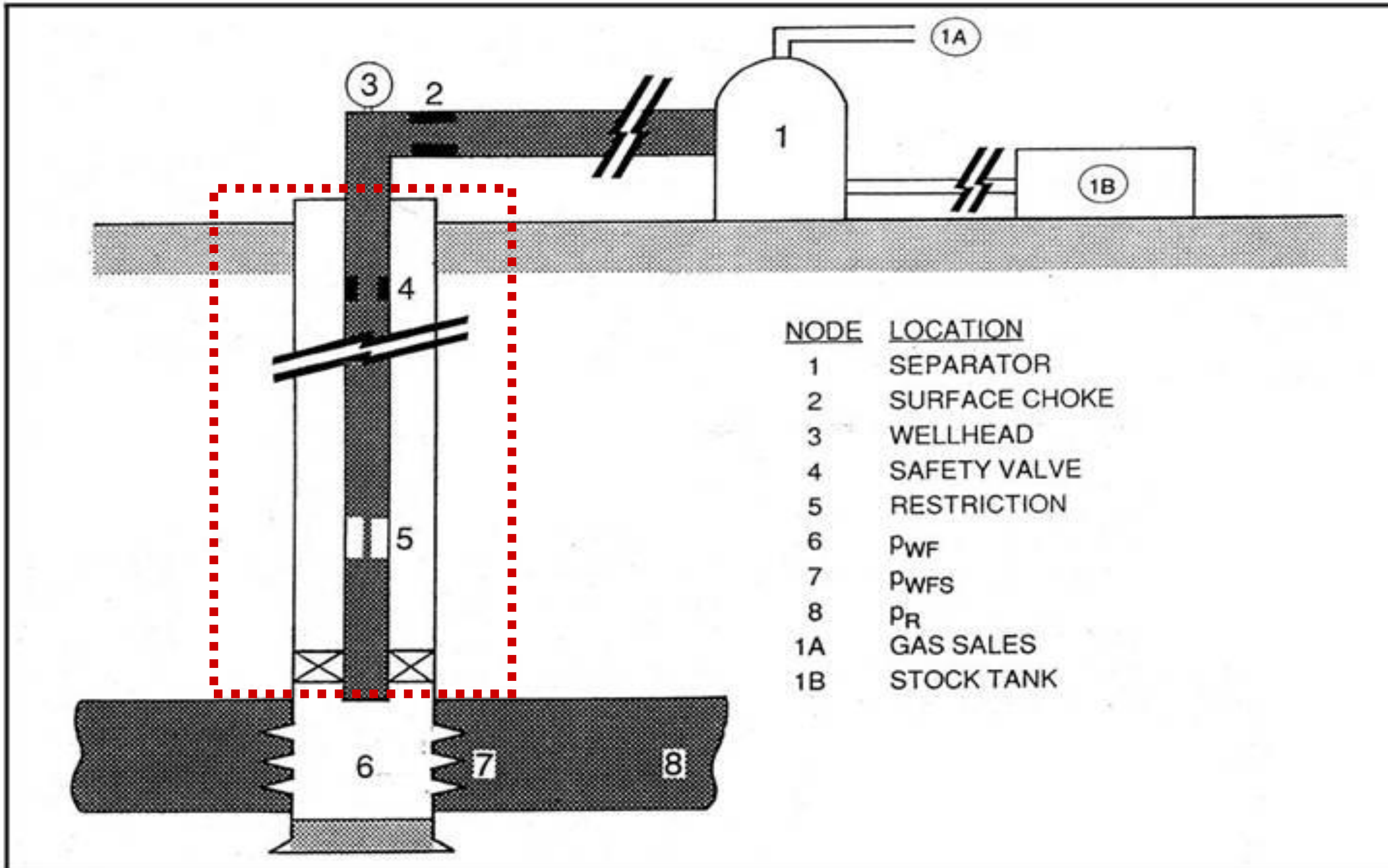
PETE 532
Outflow Well Performance

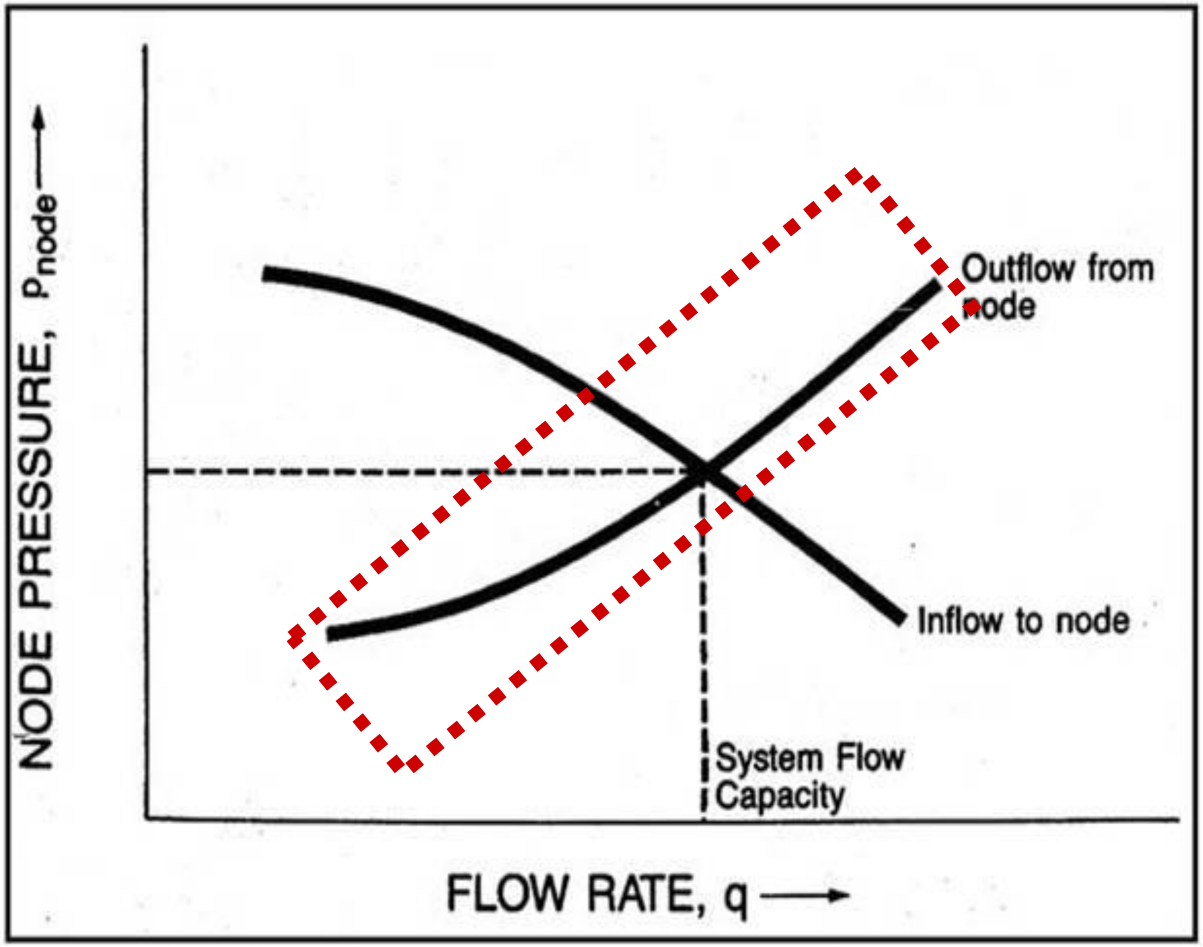
Flow in Pipes and Restrictions

Production System



Location of various nodes.

Determination of Flow Capacity

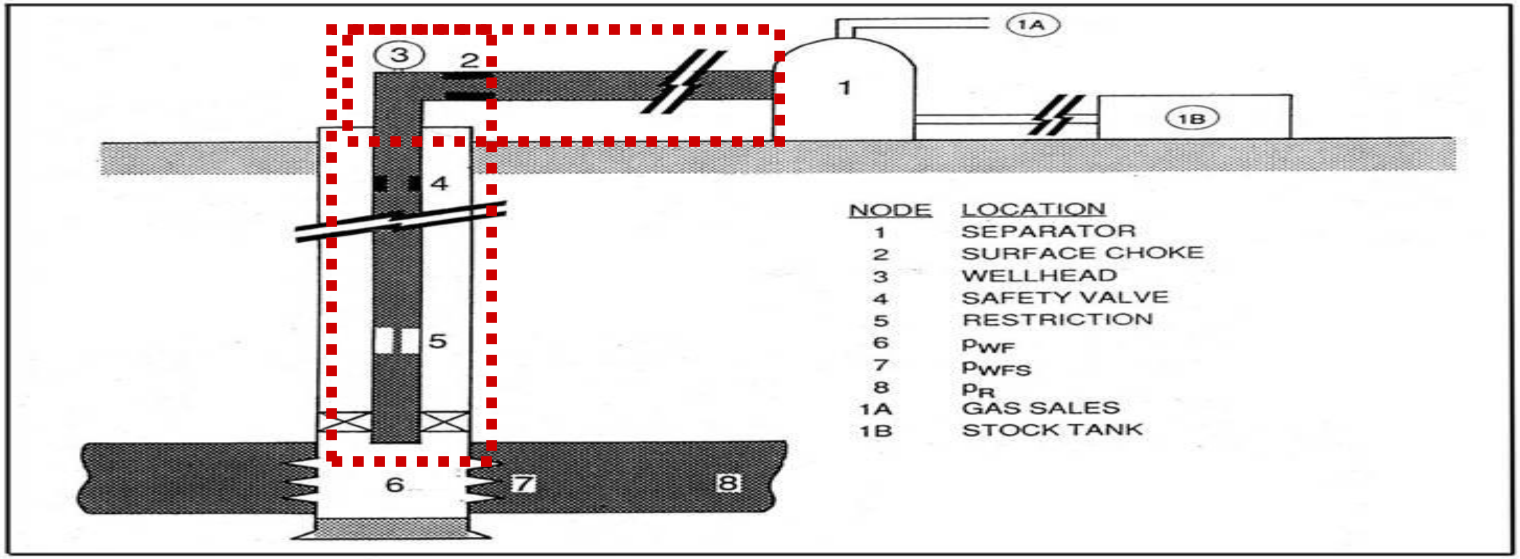


Determination of flow capacity.

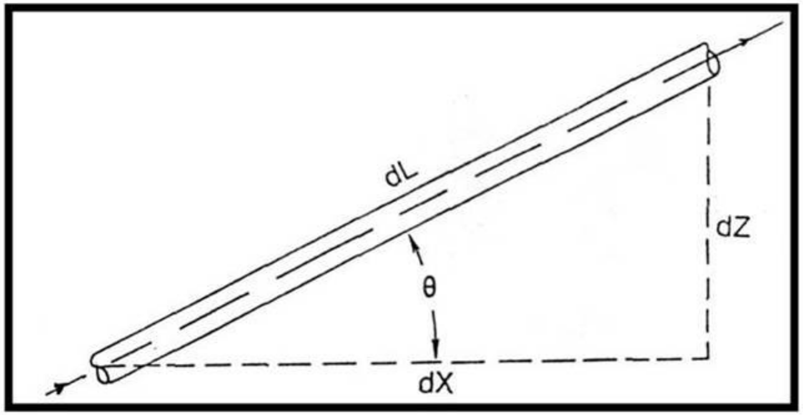
Flow Through Pipes

- Basic Equations and concept
  - General energy Equation
  - Single Phase Flow
  - Multiphase Flow
    - Liquid Holdup
    - Superficial liquid and gas velocities
    - Flow Patterns (Regimes)
    - Pressure Drop Components (friction, acceleration, head)
    - Pressure Traverse Calculations
  - PVT Calculations
  - Predicting Flowing Temperature
- Pipe Flow Correlations and Mechanistic Models

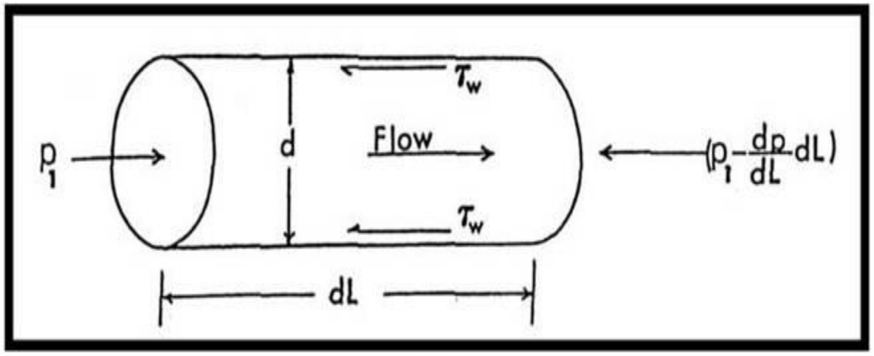
Production System



Location of various nodes.



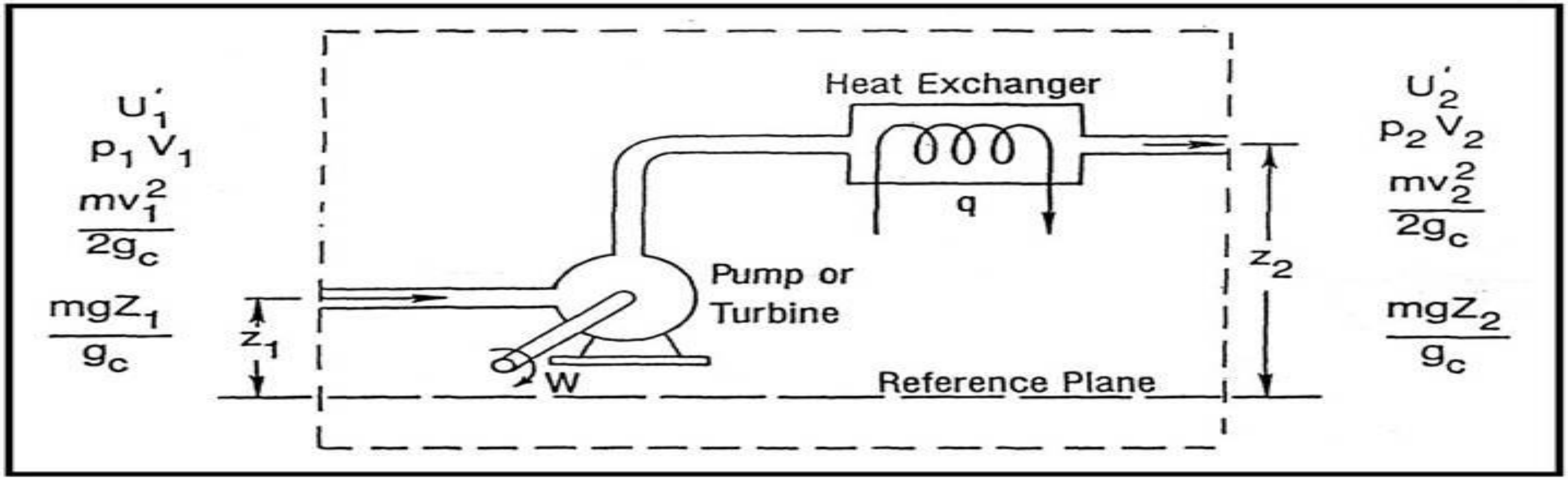
Flow geometry.



Force balance.

Initial Pressure, Shear, Gravity, Expansion

Conservation of Energy Equa



Flow system control volume.

$$\boxed{U_1' + \rho_1 V_1 + \frac{mv_1^2}{2g_c} + \frac{mgZ_1}{g_c} + q' + w_s' = U_2' + \rho_2 V_2 + \frac{mv_2^2}{2g_c} + \frac{mgZ_2}{g_c}}$$

Where:

$\frac{mv_1^2}{2g_c} =$ **Energy of Expansion**

$U_1' =$ **Internal Energy**

$\rho V =$ **Kinetic Energy**

$\frac{mgZ_1}{g_c} =$ **Potential Energy**

$q' =$ **Heat Energy added**

$w_s' =$ **Work Done**

Conservation of Energy Equa

$$U_1^{'} + \rho_1 V_1 + \frac{m v_1^{2}}{2 g_c} + \frac{m g Z_1}{g_c} + q^{'} + w_s^{'} = U_2^{'} + \rho_2 V_2 + \frac{m v_2^{2}}{2 g_c} + \frac{m g Z_2}{g_c}$$

Dividing by m to obtain energy per mass and rewriting the equation in a differential form:

$$dU + d\left(\frac{p}{\rho}\right) + \frac{v\,dv}{g_c} + \frac{g}{g_c} dZ + dq + dW_s = 0$$

Using the following thermodynamic relations to convert this energy to mechanical equation:

Conservation of Energy Equa

$$dU = dh - d(\frac{p}{\rho})$$

$$dh = TdS + \frac{dp}{\rho}$$

$$dU = TdS + \frac{dp}{\rho} - d(\frac{p}{\rho})$$

Where:
h = enthalpy
S = entropy
T = Temperature

We get:

$$TdS + \frac{dp}{\rho} + \frac{vdv}{g_c} + \frac{g}{g_c}dZ + dq + dW_s = 0$$

For irreversible process and assuming no work done on the system:

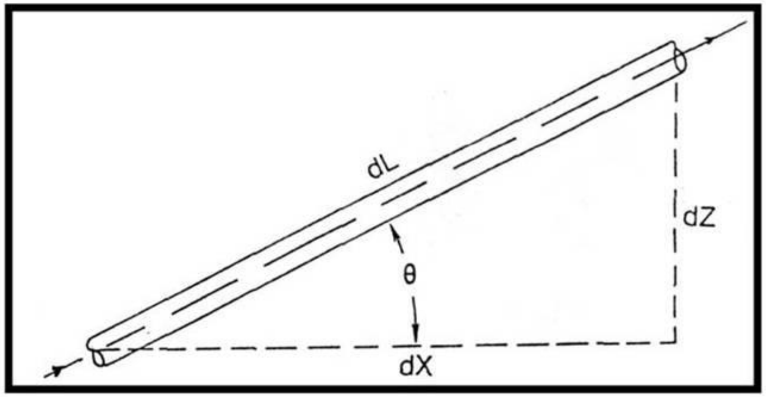
$$TdS = -dq + dL_w$$

$$\frac{dp}{\rho} + \frac{vdv}{g_c} + \frac{g}{g_c}dZ + dL_w = 0$$

Steady State = d_q=0

Flow Geometry

If we consider a pipe at an inclined angle of θ



Flow geometry.

$$dZ = dL \sin\theta$$

$$\frac{dp}{\rho} + \frac{v\,dv}{g_c} + \frac{g}{g_c} dL \sin\theta + dL_w = 0$$

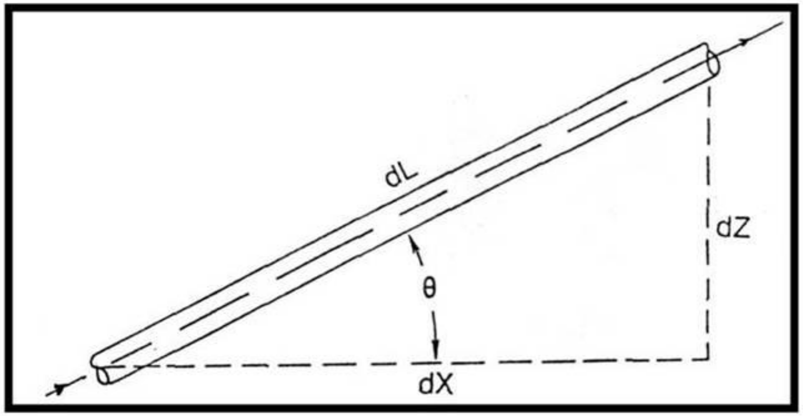
Multiplying by: $\dfrac{\rho}{dL}$

$$\frac{dp}{dL} + \frac{\rho v\,dv}{g_c dL} + \frac{g}{g_c}\rho \sin\theta + \rho\frac{dL_w}{dL} = 0$$

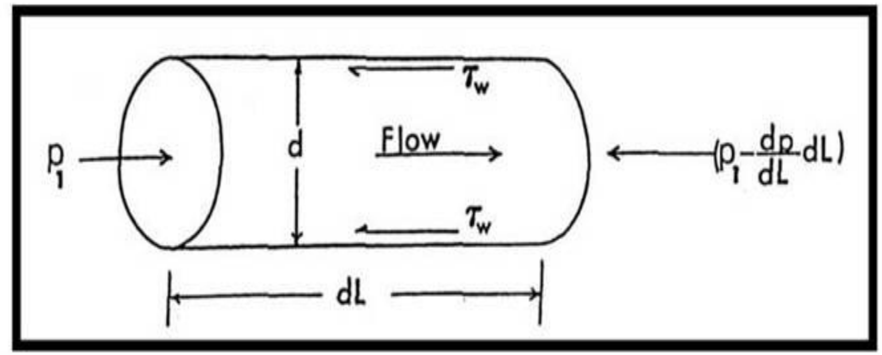
Solving for Pressure Drop:

$$\frac{dp}{dL} = \frac{\rho v\,dv}{g_c dL} + \frac{g}{g_c}\rho \sin\theta + \left(\frac{dp}{dL}\right)_f$$

Flow through Pipes



Flow geometry.



Force balance.

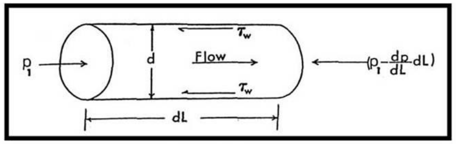
$$\frac{dp}{dL} = \boxed{\frac{\rho v\, dv}{g_c\, dL}} + \boxed{\frac{g}{g_c}\rho \sin\theta} + \boxed{\left(\frac{dp}{dL}\right)_f}$$

Acceleration Elevation Friction

Flow through Pipes

$$\frac{dp}{dL} = \frac{\rho v dv}{g_c dL} + \frac{g}{g_c}\rho \sin\theta + \boxed{\left(\frac{dp}{dL}\right)_f}$$

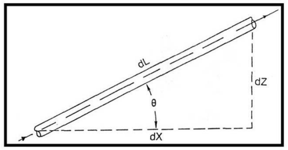
Horizontal Pipe With no change in Diameter:



Force balance.

$$\frac{dp}{dL} = \boxed{\left(\frac{dp}{dL}\right)_f}$$

Inclined or Vertical Pipe With no change in Diameter:



Flow geometry.

$$\frac{dp}{dL} = \frac{g}{g_c}\rho \sin\theta + \boxed{\left(\frac{dp}{dL}\right)_f}$$

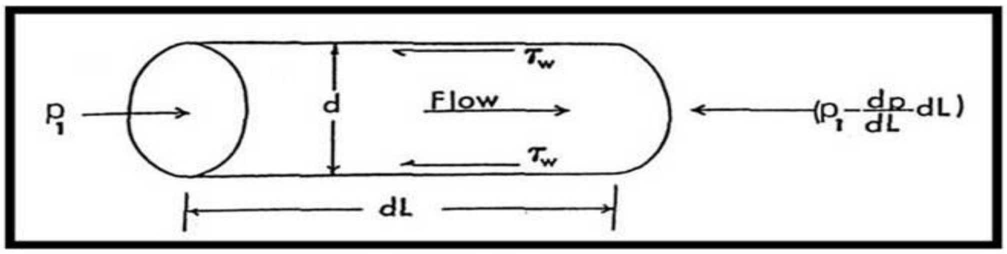
Pressure Drop Due to Friction

- In Horizontal Pipes, the energy looses or pressure drop are caused by change in kinetic energy & friction only (viscous shear).
- Since most of the viscous shear occurs at the pipe wall, the ratio of wall shear stress τ_w to kinetic energy per unit volume $\left(\frac{\rho v^2}{2g_c}\right)$ reflects the relative importance of wall shear stress to the total losses.
- This ratio forms a dimensionless group and defines a friction factor as:

$$\boxed{f' = \frac{\tau_w}{\rho v^2 / 2g_c} = \frac{2\tau_w g_c}{\rho v^2}}$$

Pressure Drop Due to Friction

- To evaluate the wall shear stress, a force balance between pressure forces and wall shear stress can be formed as follow:



Force balance.

$$\left[p_1 - \left(p_1 - \frac{dp}{dL} dL\right)\right] \frac{\pi d^2}{4} = \tau_w (\pi d) dL$$

Or:

$$\tau_w = \frac{d}{4} \left(\frac{dp}{dL}\right)_f$$

- Substituting this into $$f' = \frac{\tau_w}{\rho v^2 / 2 g_c} = \frac{2 \tau_w g_c}{\rho v^2}$$

- Solving for pressure gradient due to friction gives this well known Fanning equation. $$\left(\frac{dp}{dL}\right)_f = \frac{2 f' \rho v^2}{g_c d}$$

- Darcy-Wiesbach or Moody friction factor $f = 4 f'$ gives:

$$\left(\frac{dp}{dL}\right)_f = \frac{f \rho v^2}{2 g_c d}$$

The friction factor equation

Pressure Drop Due to Friction

- The friction factor also is a function of the type of flow. We have two types of flow: Laminar & Turbulent

Laminar Flow if Reynolds Number: $N_{Re} = (\frac{\rho v d}{\mu}) \leq 2100$

Turbulent Flow if Reynolds Number: $N_{Re} = (\frac{\rho v d}{\mu}) > 2100$

- The friction factor for Laminar flow can be determined analytically by combining Darcy-Wiesbach equation with Hagen-Poiseuille equation:

$$\left(\frac{dp}{dL}\right)_f = \frac{f \rho v^2}{2 g_c d}$$ and $$v = \frac{d^2 g_c}{32\mu}(\frac{dp}{dL})_f$$ gives: $$(\frac{dp}{dL})_f = \frac{32 \mu v}{g_c d^2}$$

- Equating both friction factor equations gives: $$\frac{32 \mu v}{g_c d^2} = \frac{f \rho v^2}{2 g_c d}$$

Or: $$f = \frac{64\mu}{\rho v d} = \frac{64}{N_{Re}}$$ Friction Factor for Laminar Flow

Friction Factor for Turbulent

- For Turbulent Flow: Due to the complexity of the physical phenomena, only empirical equation can be used to determine the friction factor for turbulent flow.

- The friction factor is sensitive to the velocity profile which is also very sensitive to the wall roughness.

- Simple imperial equations have been developed with the assumption of smooth pipe (big concern!).

- Colebrook developed the most and famous used equation for friction factor for rough pipes back in 1939:

$$\boxed{\frac{1}{\sqrt{f}} = 1.74 - 2\log\left(\frac{2\varepsilon}{d} + \frac{18.7}{N_{\mathrm{Re}}\sqrt{f}}\right)}$$

How to solve this?

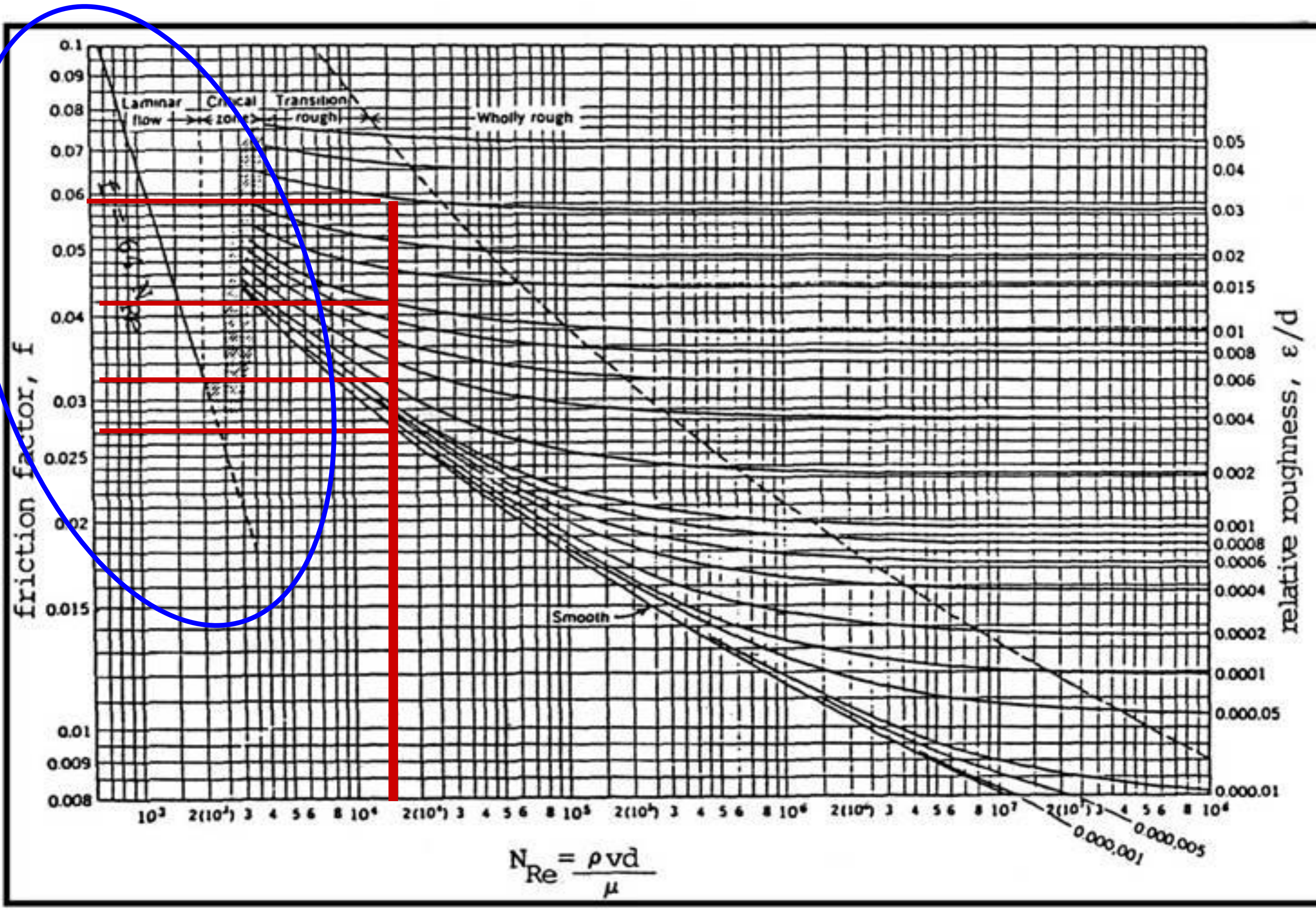
- Jain proposed an explicit equation to solve for the friction factor:

$$\boxed{\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{d} + \frac{21.25}{N_{\mathrm{Re}}^{\ 0.9}}\right)}$$

This can be used as first guess in Colebrook iterative method

He found that this equation will give results with 1% error as compared to the Colebrook equation for a wide range of pipe roughness and Reynolds number

Moody Friction Factor Diagram



Friction factors for pipe flow.[5]

Pipe Roughness

$\mathcal{E}$ Is not a property that can be physically measured.

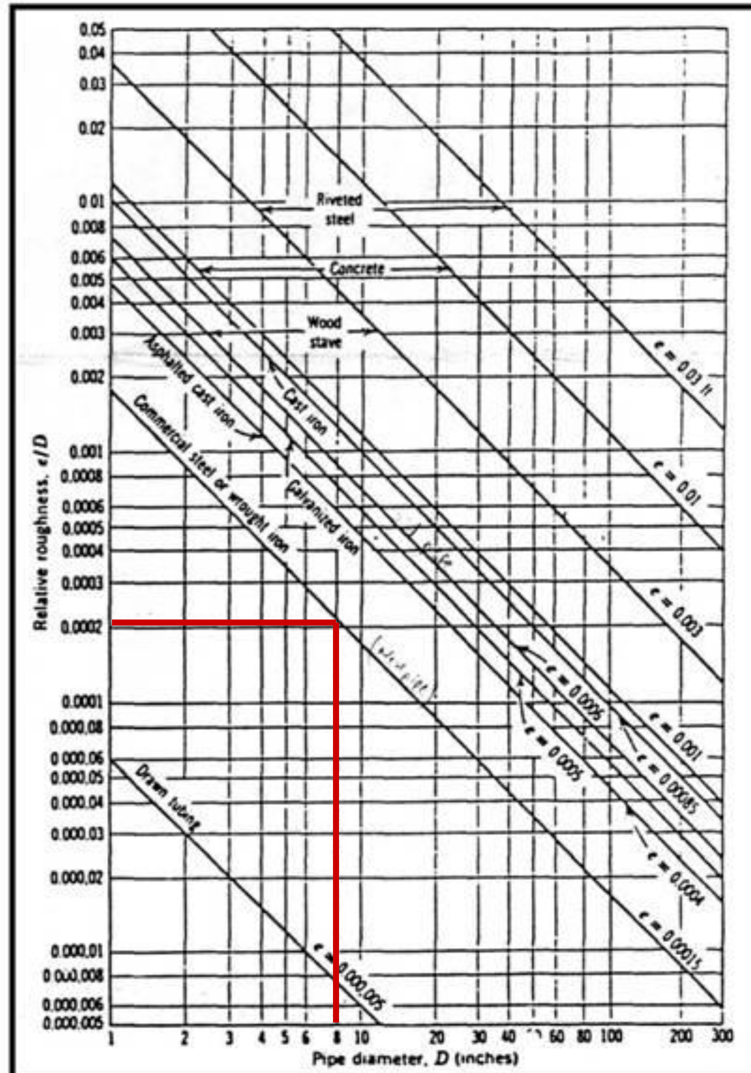
It is the sand grain roughness that would result in the same friction factor.

The only way this can be evaluated is by comparison of the behavior of a normal pipe with one that is sand roughened.

Moody has done this as shown in this figure:

These values can change significantly based on the well/pipe conditions such as paraffin, corrosion, erosion scale deposition, etc.

If we have measured pressure gradient, we can back calculate it and then use it for future



Pipe roughness.[5]

Friction Factor Example

A liquid of specific gravity 0.82 and viscosity of 3cp flows in a 4in (0.1016 m) New commercial pipe at a velocity of 30 ft/sec (9.14 m/sec).
Calculate the friction factor using Colebrook and Jain equations.

Answer: Determine the flow type:

$\boxed{N_{\mathrm{Re}} = (\frac{\rho v d}{\mu})}$ = 820*9.14*(0.1016)/0.003=253,824 Flow is turbulent.

$\boxed{\varepsilon/d =}$ 0.00045

Jain friction:

$$\boxed{\frac{1}{\sqrt{f}} = 1.14 - 2\log(\frac{\varepsilon}{d} + \frac{21.25}{N_{\mathrm{Re}}^{0.9}})} = 0.0182$$

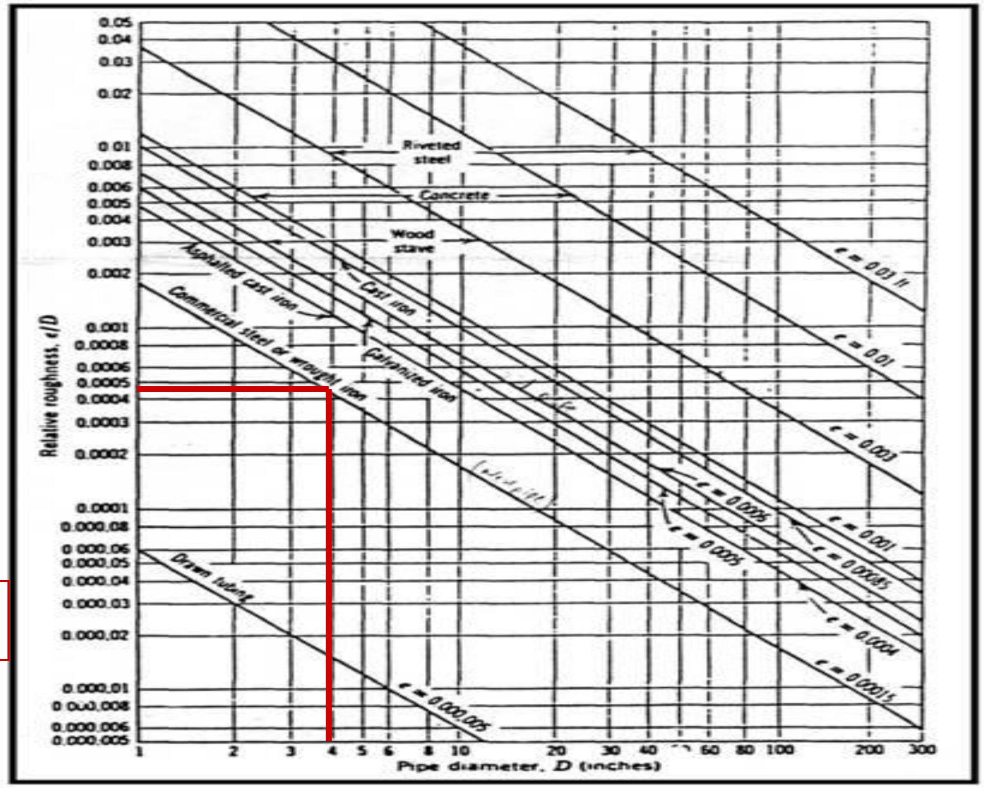
Colebrook friction:

$$\boxed{\frac{1}{\sqrt{f}} = 1.74 - 2\log(\frac{2\varepsilon}{d} + \frac{18.7}{N_{\mathrm{Re}}\sqrt{f}})}$$

F_1= 0.0182

F_{c1}= 0.0181 Is it good enough?

F_{c2}= 0.0181



Pipe roughness.[5]

Single Phase Pressure Drop Example

Calculate the pressure drop that occurs in a 200 m section of 3.94 in Hz pipe when a liquid having a viscosity of 50 cp and density of 900 kg/m^3 flows at a rate of 0.135 ft^3/sec & rate of 0.83 ft^3/sec

$$\frac{dp}{dL} = \cancel{\frac{\rho v dv}{g_c dL}} + \cancel{\frac{g}{g_c}\rho} \sin\theta + \left(\frac{dp}{dL}\right)_f$$

$$\left(\frac{dp}{dL}\right)_f = \frac{f\rho v^2}{2 g_c d}$$

$$f = \frac{64\mu}{\rho v d} = \frac{64}{N_{\mathrm{Re}}}$$

or

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{d} + \frac{21.25}{N_{\mathrm{Re}}^{0.9}}\right)$$

$$N_{\mathrm{Re}} = \left(\frac{\rho v d}{\mu}\right)$$

Case 1: Reynolds Number= 900 Flow is Laminar

Friction: $f = \frac{64\mu}{\rho v d} = \frac{64}{N_{\mathrm{Re}}}$ = 0.0711

Pressure drop: $dp = \frac{f\rho v^2}{2 g_c d}$ * 200 = 15975 N/M^2 = 15.975 K Pa = 2.31 psi

Single Phase Flow

Case II: rate of 0.83 ft^3/sec

$$N_{\mathrm{Re}} = \left(\frac{\rho v d}{\mu}\right)$$

Reynolds Number= 5400 Flow is Turbulent

Assume $\boxed{\varepsilon}$ = 0.0006 ft

Using Jain equation: $\boxed{\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{d} + \frac{21.25}{N_{\mathrm{Re}}^{0.9}}\right)}$

Friction: = 0.0393

Pressure drop: $\left(\frac{dp}{dL}\right)_f = \frac{f \rho v^2}{2 g_c d}$ * 200 = 318300 N/M^2 = 318.3 K Pa = 46.2 psi

What would happen if the pipe is vertical (Producer)?

$$\frac{dp}{dL} = \frac{\rho v\, dv}{g_c\, dL} + \frac{g}{g_c} \rho \sin\theta + \left(\frac{dp}{dL}\right)_f$$

What would happen if the pipe is vertical (Injector)?

What would happen if the pipe is inclined at 45 degree?

Summary of Single Phase Flow

$$\frac{dp}{dL} = \frac{\rho v dv}{g_c dL} + \frac{g}{g_c}\rho \sin\theta + \frac{f \rho v^2}{2 g_c d}$$

$$\left(\frac{dp}{dL}\right)_f = \frac{f \rho v^2}{2 g_c d}$$

$$N_{\mathrm{Re}} = \left(\frac{\rho v d}{\mu}\right)$$

<2100

$$f = \frac{64}{N_{\mathrm{Re}}}$$

>2100

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{d} + \frac{21.25}{N_{\mathrm{Re}}^{0.9}}\right)$$

or

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\left(\frac{2\varepsilon}{d} + \frac{18.7}{N_{\mathrm{Re}}\sqrt{f}}\right)$$

Oil Field Units

$$\rho = lbm / ft^3$$

$$v = ft/\sec$$

$$d, l, \varepsilon = ft$$

$$\mu = lb / ft - \sec = 1488 cp$$

$$g, g_c = 32.2$$

$$\left(\frac{dp}{dL}\right)_f = psf / ft = (1/144) psi / ft$$

Summary of Single Phase Flow

Metric Field Units

$$\frac{dp}{dL} = \frac{\rho v dv}{g_c dL} + \frac{g}{g_c}\rho \sin\theta + \left(\frac{f\rho v^2}{2g_c d}\right)$$

$$Gravity = \frac{g}{g_c}\rho\sin\theta = \frac{32.2}{32.2}\times\rho\left(\frac{lb}{ft^3}\times\frac{1\,ft^2}{144in^2}\right)\times\sin\theta^o = psi/ft$$

Friction:

$$N_{Re} = \frac{\rho v d}{\mu}$$

$$v = \frac{q}{A} = \frac{q(B/D)}{\pi/4(D(in)/12)^2\, ft^2}\times\frac{5.615\,ft^3}{1B}\times\frac{1D}{24hr}\times\frac{1hr}{60\min}\times\frac{1\min}{60\sec} = ft/\sec$$

$$\rho = 62.4lb/ft^3 = 1000kg/m^3$$

$$\therefore N_{Re} = \frac{\rho(kg/m^3)\times v(\frac{ft}{\sec})\times\frac{1m}{3.28\,ft}\times D(in)\times\frac{1\,ft}{12in}\times\frac{1m}{3.28\,ft}}{\mu(Poise)} = Dimensionless$$

$$\frac{\varepsilon}{d} = \frac{\varepsilon}{D(in)/12} = Dimensionless$$

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{d} + \frac{21.25}{N_{Re}^{0.9}}\right)$$

$$\left(\frac{dp}{dL}\right)_f = \frac{f\times\rho(Ib/ft^3)\times v^2(ft/\sec)}{2\times 32.2\times D(in)/12} = f\left(\frac{lb}{ft^3}\right)\times\frac{1\,ft^2}{144in^2} = f(psi/ft)$$

$$dp = \frac{dp}{dL}(psi/ft)\times L(ft) = dp(psi)$$

Summary of Single Phase Flow

Simplified Formulas in Field Units

$$\frac{dp}{dL} = \frac{\rho v dv}{g_c dL} + \frac{g}{g_c}\rho \sin\theta + \left(\frac{f\rho v^2}{2 g_c d}\right)_f$$

$$\Delta P_{Gravity}(psi) = 0.433 * SG * L * \sin\theta^o$$

Units:
Pressure = psi
Diameter = in
Rate = Bl/D
Length = ft
Viscosity = CP
P Roughness = ft
SG = Density(lb/ft^3)/62.4
ε = in

Friction:

$$\Delta P_f(psi) = 0.000011466 * f * L * \frac{q^2}{d^5}$$

$$v(ft/\sec) = 0.0119175 * \frac{q}{d^2}$$

$$N_{RE} = 92.16 * \frac{q}{d} * \frac{SG}{\mu}$$

<2100 $\quad f = \frac{64}{N_{Re}}$

>2100 $\quad \frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{d} + \frac{21.25}{N_{Re}^{0.9}}\right)$

Homework#4
Due day 3/27/2011

Calculate your injectivity index in a vertical water injector with the following data:

Injection P = 1000 PSI

Reservoir Pressure = 3000 PSI

Depth=8000 ft

Injection rate = 20,000 bbl/D

Density= 62.4 lbm/ft^3

Tubing Diameter = 5 in.

Viscosity = 1 cp

ε **=0.0006 ft**